# Macroeconomics B Exam

Exam Number 29

14. juni 2024

# 1 Problem 1

## 1.1

**Equation 1**
Describes the equilibrium in the goods market. The output gap is a negative function of deviations from the natural interest rate ($\bar{r}$). Thus the goods market is in an equilibrium when the real interest rate is equal to the natural interest rate. The model also states that any shocks to consumer or business confidence will affect the output.

$$y_t - \bar{y} = \hat{y}_t\left(\underset{-}{r_t}, \underset{+}{\bar{r}}, \underset{+}{v_t}\right)$$

**Equation 2**
This equation describes the ex-ante real interest rate and is referred to as the Fisher equation. The real interest is given by subtracting the expected inflation for period $t+1$ from the nominal interest rate at time $t$. Thus it is an expression of how the expected real interest rate is to be.

$$r_t\left(\underset{+}{i_t}, \underset{-}{\pi^e_{t+1,t}}\right)$$

**Equation 3**
The Taylor rule, states how the central bank conducts its monetary policy, thus how it sets the nominal interest rate $i_t$. We see that the baseline of the nominal interest rate is $\bar{r} + \pi^e_{t+1,t}$ and that there is a part that the central bank can't control, but still affects the nominal interest rate $\hat{\rho}_t$. The central bank reacts to both changes in inflation and output gaps. The outputs gap is however with an error term $\epsilon_t$, since it is often not possible to 100% measure the output of the current period. $h$ describes to what magnitude to which the central bank reacts to changes in inflation from the target inflation $\pi^*$. Whereas $b$ is how strongly it reacts to an output gap.

$$i_t\left(\underset{+}{\bar{r}}, \underset{+}{\pi^e_{t+1,t}}, \underset{+}{\pi_t}, \underset{-}{\pi^*}, \underset{+}{y_t}, \underset{+}{\epsilon_t}, \underset{-}{\bar{y}}, \underset{+}{\hat{\rho}_t}\right)$$

**Equation 4**
The short-run aggregate supply curve (SRAS) shows that the inflation rate for the period $t$ is determined by the expected inflation for this period, which was forecasted in the previous period $t-1$ ($\pi^e_{t,t-1}$), along with the output gap at $t$ multiplied by $\gamma$, which indicates the sensitivity of inflation to output gaps.
A higher output level is typically linked to increased employment levels. Given the diminishing marginal product of labor, as firms hire more workers, the marginal cost of labor rises. This leads to higher prices and therefore more inflation. Additionally, higher expected inflation makes unions demand higher nominal wages, which further increases firms' marginal costs and thus again higher inflation.

$$\pi_t\left(\underset{+}{\pi^e_{t,t-1}}, \underset{+}{y_t}, \underset{-}{\bar{y}}\right)$$

## 1.2

To derive the AD curve for the economy, we can simply substitute equation (3) into (2), and then that into equation (1). $\pi^e_{t,t-1}$ and $\bar{r}$ will cancel out, thus giving

$$y_t - \bar{y} = -\alpha_2(h(\pi_t - \pi^*) + b(y_t + \epsilon_t - \bar{y}) + \hat{\rho}_t) + v_t \tag{1}$$

From there simply isolate the output gap and giving

$$y_t - \bar{y} = -\alpha_2(h(\pi_t - \pi^*) + b(y_t + \epsilon_t - \bar{y}) + \hat{\rho}_t) + v_t$$

$$\Longleftrightarrow$$

$$y_t - \bar{y} = -\alpha_2 h(\pi_t - \pi^*) - \alpha_2 b(y_t + \epsilon_t - \bar{y} + \hat{\rho}_t) + v_t$$

$$\Longleftrightarrow$$

$$y_t - \bar{y} + \alpha_2 b(y_t + \epsilon_t - \bar{y} + \hat{\rho}_t) = -\alpha_2 h(\pi_t - \pi^*) + v_t$$

$$\Longleftrightarrow$$

$$y_t - \bar{y}(1 + \alpha_2 b) = -\alpha_2 h(\pi_t - \pi^*) + v_t$$

$$\Longleftrightarrow$$

$$y_t - \bar{y} = -a(\pi_t - \pi^*) + z$$

Where $z = (-\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t)\frac{1}{1+\alpha_2 b}$ and $a = \frac{\alpha_2 h}{1+\alpha_2 b}$.

We can analyze the slope by deriving the AD curve with regard to inflation such as

$$\frac{\partial y_t}{\partial \pi_t} = -\frac{\alpha_2 h}{1 + \alpha_2 b} \tag{2}$$

We see that the slope is negative, thus an increase in inflation will increase the AD curve and vice versa.
An increase in $h$ will make the slope steeper since it is given in the numerator of the slope. Due to the central bank reacting more strongly to changes in inflation.
An increase in $b$ will make the slope flatter since it is given in the denominator. Due to the central bank reacting more strongly to changes in output gaps.
When looking at $\alpha_2$ it is both in the nominator and denominator, thus we can not simply say which direction it pulls in as it depends on $h$ and $b$. We will see a steeper curve if $h > b$, and vice versa a flatter curve if $b < h$, and unchanged if $h = b$.

## 1.3

Given an increase in measurement error in periods 1 and 2, we can find the magnitude and the direction by isolating $\pi_t$ and differentiating with regards to $\epsilon_t$:

$$y_t - \bar{y} = -a(\pi_t - \pi^*) + (-\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t)\frac{1}{1+\alpha_2 b}$$

$$\iff$$

$$\pi_t = \frac{1}{a}((-\alpha_2(\hat{\rho} + b\epsilon_t) + v_t)\frac{1}{1+\alpha_2 b} - y - \bar{y}) + \pi^*$$

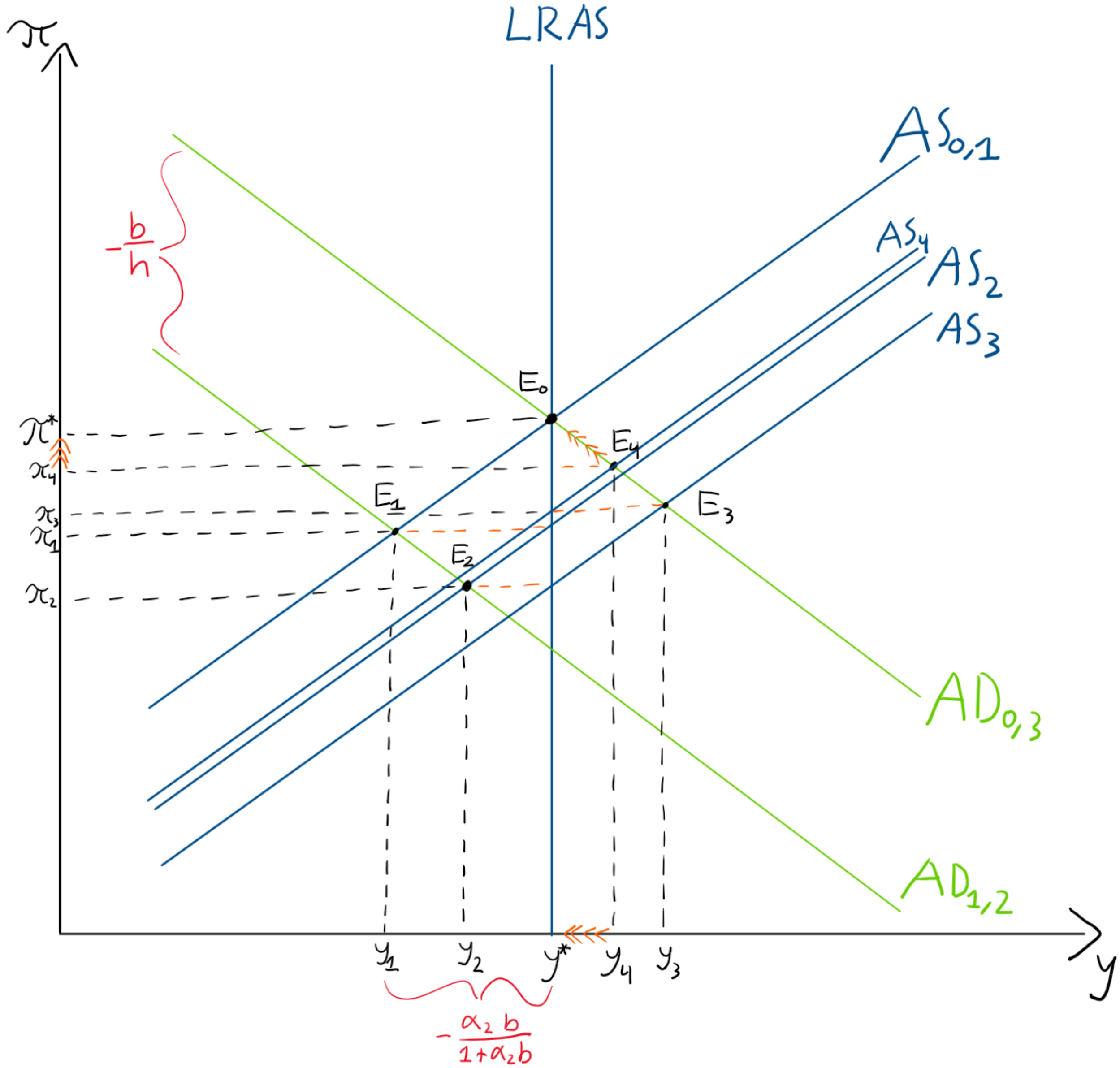
Differentiation with respect to $\epsilon_t$:

$$\frac{\partial \pi}{\partial \epsilon_t} = (\frac{-\alpha_2 b\epsilon}{1+\alpha_2 b} \cdot \frac{1}{a})' = \frac{-\alpha_2 b}{1+\alpha_2 b} \cdot \frac{1}{\frac{\alpha_2 h}{1+\alpha_2 b}} = -\frac{b}{h}$$

Thus we notice that the AD curves shifted down with a magnitude of $-\frac{b}{h}$.
We can also calculate the horizontal shift of $y$ by differentiating the AD of $y$:

$$\frac{\partial y_t}{\partial \epsilon} = -\frac{\alpha_2 b}{1+\alpha_2 b}$$

Thus we see a decrease of $-\frac{\alpha_2 b}{1+\alpha_2 b}$ in output. This can be illustrated in figure 1 when $\epsilon$ increases by 1.




Figur 1: Illustration of the measurement shock.

Thus we see that we first move the AD curve down in period 1. This will make the expected inflation for period 2 change, and thus the AS curve also moves down in period 2. In period 3 we do not have any measurement error, and the AD curve moves back. We are now out of equilibrium still, and the AS curve will then slowly converge back into the long-run equilibrium.

## 1.4

Starting from the AD curve found in 1.2 without any shocks and substituting equation (5) into (4) and that into the AD curve to give:

$$\hat{y}_t = -a(\pi_t - \pi^*)$$

$$\Longleftrightarrow$$

$$\hat{y}_t = -a(\pi_{t-1} + \gamma\hat{y}_t - \pi^*)$$

Isolate $\hat{y}_t$

$$\hat{y}_t + a\gamma\hat{y}_t = -a(\pi_{t-1} - \pi^*)$$

$$\Longleftrightarrow$$

$$\hat{y}_t(1 + a\gamma) = -a(\pi_{t-1} - \pi^*)$$

$$\Longleftrightarrow$$

$$\hat{y}_t = -\frac{a}{1 + a\gamma}(\pi_{t-1} - \pi^*)$$

Thus we need the inflation term $(\pi_{t-1} - \pi^*)$ in terms if $\hat{y}_{t-1}$, we therefore take the AD curve to the period before $(t - 1)$

$$\hat{y}_{t-1} = -a(\pi_{t-1} - \pi^*) \iff \pi_{t-1} - \pi^* = -\frac{\hat{y}_{t-1}}{a}$$

Substituting that in

$$\hat{y}_t = -\frac{a}{1 + a\gamma}(-\frac{\hat{y}_{t-1}}{a}) \iff \hat{y}_t = \frac{1}{1 + a\gamma}\hat{y}_{t-1}$$

Thus $\beta = \frac{1}{1+a\gamma} = \frac{1}{1+\frac{\alpha_2 h}{1+\alpha_2 b}\gamma} = \frac{1+\alpha_2 b}{1+\alpha_2 b+\alpha_2 h\gamma}$

The convergence towards long-run equilibrium is faster if the central bank places more emphasis on output stabilization (higher $b$) relative to inflation stabilization (lower $h$). $\alpha_2$ is determining the sensitivity of the output gap to interest rate changes.

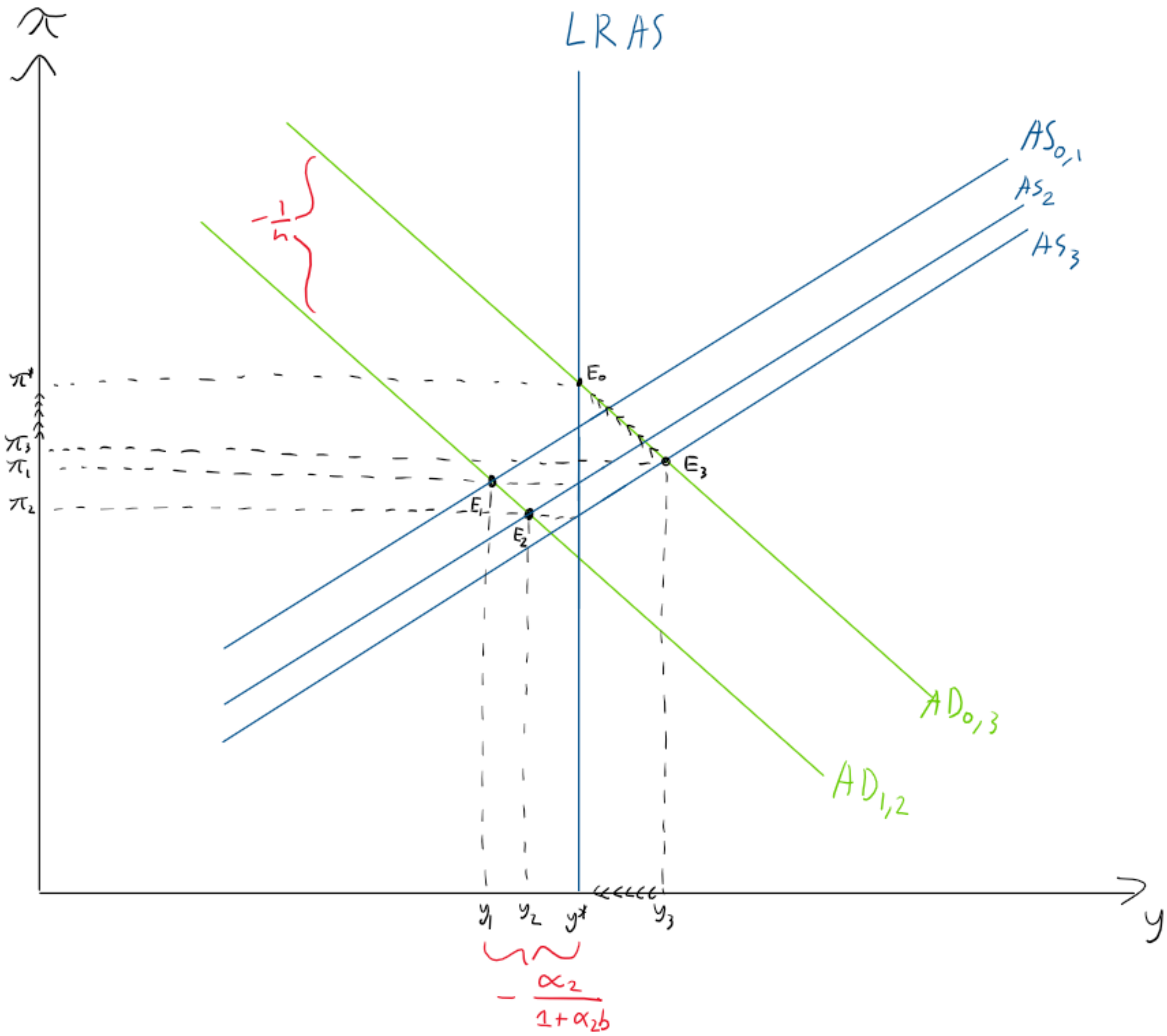
## 1.5

Using the same approach from 1.3, we can differentiate with regards to $\hat{\rho}_t$

$$\frac{\partial \pi}{\partial \hat{\rho}_t} = (\frac{-\alpha_2\hat{\rho}_t}{1 + \alpha_2 b} \cdot \frac{1}{a})' = -\frac{1}{h}$$

Thus we again notice a downward shift. Analyzing the vertical shift with the same approach again:

$$\frac{\partial y_t}{\partial \hat{\rho}_t} = -\frac{\alpha}{1 + \alpha_2 b}$$




Figur 2: Illustration of a shock of $\hat{\rho}$

We notice a shift in the same direction as question 1.3, but we see that the magnitudes are different, but the behavior is the same.

## 1.6

First, we solve for $y_t$ in the AD curve by inserting the AS curve:

$$y_t - \bar{y} = -a(\pi_t - \pi^*) + z \iff y_t - \bar{y} = -a(\pi^e_{t,t-1} + \gamma(y_t - \bar{y}) - \pi^*) + z$$

$$\iff$$

$$y_t + a\gamma y_t = \bar{y} - a(\pi^e_{t,t-1} + a\bar{y} + a\pi^*) + z$$

$$\iff$$

$$y_t = (\bar{y} - a(\pi^e_{t,t-1} - \bar{y} - \pi^*) + z)\frac{1}{1+a\gamma}$$

$$\iff$$

$$y_t = \bar{y} - \frac{a\pi^e_{t,t-1}}{1+a\gamma} + \frac{a\pi^*}{1+a\gamma} + \frac{z}{1+a\gamma} = y_t = \bar{y} - (a\pi^e_{t,t-1} - a\pi^* - z)\frac{1}{1+a\gamma}$$

And for the AS curve we insert the AD and isolate $\pi_t$:

$$\pi_t = \pi^e_{t,t-1} + \gamma(-a(\pi_t - \pi^*) + z) \iff \pi_t + \gamma a\pi_t = \pi^e_{t,t-1} + \gamma a\pi^* + \gamma z$$

$$\iff$$

$$\pi_t = (\pi^e_{t,t-1} + \gamma a\pi^* + \gamma z)\frac{1}{1+\gamma a} \iff \pi_t = \frac{\pi^e_{t,t-1} + \gamma a\pi^* + \gamma z}{1+\gamma a}$$

We can then find the expectations of the inflation based on the rational expectations:

$$\pi^e_{t,t-1} = E(\pi_t|I_{t-1}) = E\left(\frac{\pi^e_{t,t-1} + \gamma a\pi^* + \gamma z}{1+\gamma a}|I_{t-1}\right)$$

Given all shocks are in expectation 0, $z = 0$. And we then isolate $\pi^*$:

$$\pi^e_{t,t-1} = E\left(\frac{\pi^e_{t,t-1} + \gamma a\pi^* + \gamma z}{1+\gamma a}|I_{t-1}\right) \iff \pi^e_{t,t-1} = \frac{\pi^e_{t,t-1} + \gamma a\pi^*}{1+\gamma a} \iff \pi^e_{t,t-1} = \pi^*$$

Given that fact, we can now substitute $\pi^e_{t,t-1} = \pi^*$ into the equations we found earlier, and then we just need to simplify:

$$y_t = \bar{y} - (a\pi^* - a\pi^* - z)\frac{1}{1+a\gamma} = \bar{y} + (-\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t)\frac{1}{1+\alpha_2 b}\frac{1}{1+\frac{\alpha_2 h}{1+\alpha_2 b}\gamma}$$

$$= \bar{y} + (-\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t) \cdot \frac{1}{1+\alpha_2 b + \alpha_2 h\gamma} = \bar{y} + \frac{-\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t}{1+\alpha_2 b + \alpha_2 h\gamma}$$

$$= -\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t = v_t - \alpha_2(\hat{\rho}_t + b\epsilon_t) = \bar{y} + \frac{v_t - \alpha_2(\hat{\rho}_t + b\epsilon_t)}{1+\alpha_2 b + \alpha_2 h\gamma}$$

$$= \bar{y} + \frac{v_t - \alpha_2 b\epsilon_t - \alpha_2\hat{\rho}_t}{1+\alpha_2 b + \gamma\alpha_2 h} \tag{3}$$

And for $\pi_t$:

$$\pi_t = \frac{\pi^* + \gamma\frac{\alpha_2 h}{1+\alpha_2 b}\pi^* + \gamma(-\alpha_2(\hat{\rho}_t + b\epsilon_t) + v_t)\frac{1}{1+\alpha_2 b}}{1+\gamma\frac{\alpha_2 h}{1+\alpha_2 b}} = \frac{(1+\alpha_2 b + \gamma\alpha_2 h)\pi^* + \gamma v_t - \gamma\alpha_2(\hat{\rho}_t + b\epsilon_t)}{1+\gamma\frac{\alpha_2 h}{1+\alpha_2 b}}$$

$$= \frac{(1+\alpha_2 b + \gamma\alpha_2 h)\pi^* + \gamma v_t - \gamma\alpha_2(\hat{\rho}_t + b\epsilon_t)}{1+\alpha_2 b + \gamma\alpha_2 h} = \pi^* + \gamma\frac{v_t - \alpha_2(\hat{\rho}_t + b\epsilon_t)}{1+\alpha_2 b + \gamma\alpha_2 h} = \pi^* + \gamma\frac{v_t - \alpha_2 b\varepsilon_t - \alpha_2\hat{\rho}_t}{1+\alpha_2 b + \gamma\alpha_2 h}$$

$$= \pi^* + \gamma\frac{v_t - \alpha_2 b\epsilon_t - \alpha_2\hat{\rho}_t}{1+\alpha_2 b + \gamma\alpha_2 h} \tag{4}$$

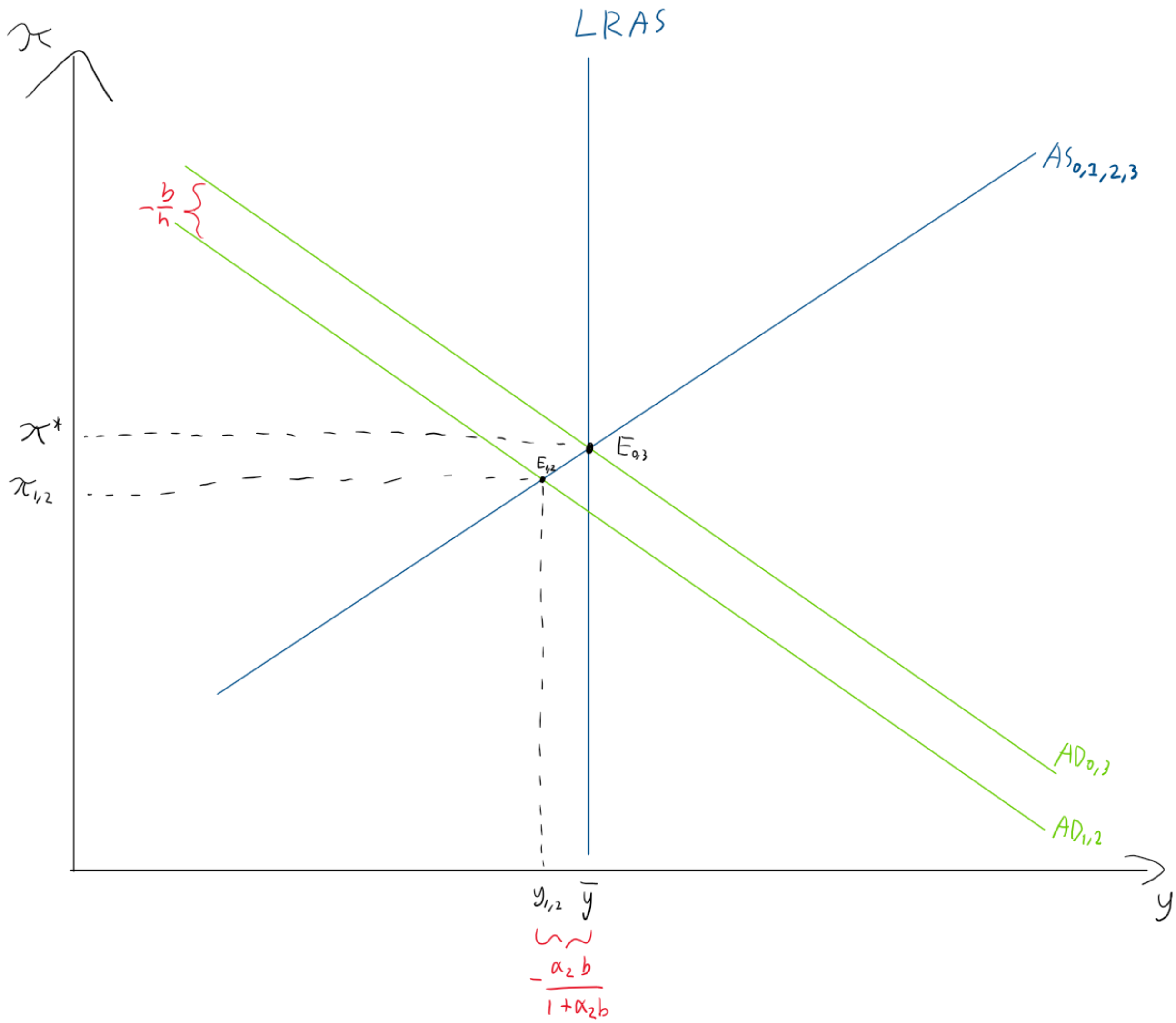
Thus the equilibrium of output and inflation can be given by (3) and (4).

## 1.7

We see that from equations (3) and (4), policy parameters $b$ and $h$ are apparent, thus the inflation and output are affected by the monetary policy. Therefore PIP breaks down, since the central bank indeed can affect how the real economic variables.

## 1.8

We again have the same shift as in question 3 for the AD curve, thus the AD curve shifts in the same manner and magnitude as in question 3. However, we have that the expected inflation follows $\pi^e_{t,t-1} = \pi^*$, and therefore our AS curve does not change. Thus the equilibrium is the same for question 3 in periods $t = 0$ and $t = 1$, but for $t = 2$ we now stay in the same equilibrium as in $t = 1$, but for $t = 3$ we move straight back to the original equilibrium. See illustrated below in the figure 3.




Figur 3: Illustration of shift given a shock to error measurement with static expectations.

## 1.9

Given:

$$\sigma_y^2 = \frac{\sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2 \sigma_\rho^2}{(1 + \alpha_2 b + \gamma \alpha_2 h)^2}$$

We differentiate $\sigma_y^2$ with respect to $b$ using the quotient rule:

$$\frac{\partial \sigma_y^2}{\partial b} = \frac{(2\alpha_2^2 b \sigma_\varepsilon^2)(1 + \alpha_2 b + \gamma \alpha_2 h)^2 - \left(\sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2 \sigma_\rho^2\right) \cdot 2(1 + \alpha_2 b + \gamma \alpha_2 h) \cdot \alpha_2}{(1 + \alpha_2 b + \gamma \alpha_2 h)^4}$$

$$= \frac{2\alpha_2^2 b\sigma_\varepsilon^2 (1+\alpha_2 b+\gamma\alpha_2 h) - 2\alpha_2 \left(\sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2\sigma_\rho^2\right)}{(1+\alpha_2 b+\gamma\alpha_2 h)^3}$$
$$= \frac{2\alpha_2 \left[\alpha_2 b\sigma_\varepsilon^2 (1+\alpha_2 b+\gamma\alpha_2 h) - \left(\sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2\sigma_\rho^2\right)\right]}{(1+\alpha_2 b+\gamma\alpha_2 h)^3}$$

Set equal to 0 and then just solve for b:

$$2\alpha_2 \left[\alpha_2 b\sigma_\varepsilon^2 (1+\alpha_2 b+\gamma\alpha_2 h) - \left(\sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2\sigma_\rho^2\right)\right] = 0$$
$$\Longleftrightarrow$$
$$\alpha_2 b\sigma_\varepsilon^2 (1+\alpha_2 b+\gamma\alpha_2 h) = \sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2\sigma_\rho^2$$

$$\Longleftrightarrow$$

$$\alpha_2 b\sigma_\varepsilon^2 + \alpha_2^2 b^2\sigma_\varepsilon^2 + \alpha_2 b\gamma\alpha_2 h\sigma_\varepsilon^2 = \sigma_v^2 + \alpha_2^2 b^2 \sigma_\varepsilon^2 + \alpha_2^2\sigma_\rho^2$$

$$\Longleftrightarrow$$

$$\alpha_2 b\sigma_\varepsilon^2 + \alpha_2 b\gamma\alpha_2 h\sigma_\varepsilon^2 = \sigma_v^2 + \alpha_2^2\sigma_\rho^2 \iff \alpha_2 b\sigma_\varepsilon^2 (1+\gamma\alpha_2 h) = \sigma_v^2 + \alpha_2^2\sigma_\rho^2$$

$$\Longleftrightarrow$$

$$b = \frac{\sigma_v^2 + \alpha_2^2\sigma_\rho^2}{\alpha_2\sigma_\varepsilon^2 (1+\gamma\alpha_2 h)}$$

## 1.10

The optimal value of $b$ minimizes the variance of the output ($\sigma_y^2$). The variance of the shocks ($\sigma_\epsilon^2$, $\sigma_\rho^2$, and $\sigma_v^2$) indicates the magnitude of potential disturbances. If a shock has a high variance, we expect larger fluctuations when such shocks occur. Therefore, the optimal $b$ reflects a trade-off based on these variances and the central bank's reaction parameter $h$.

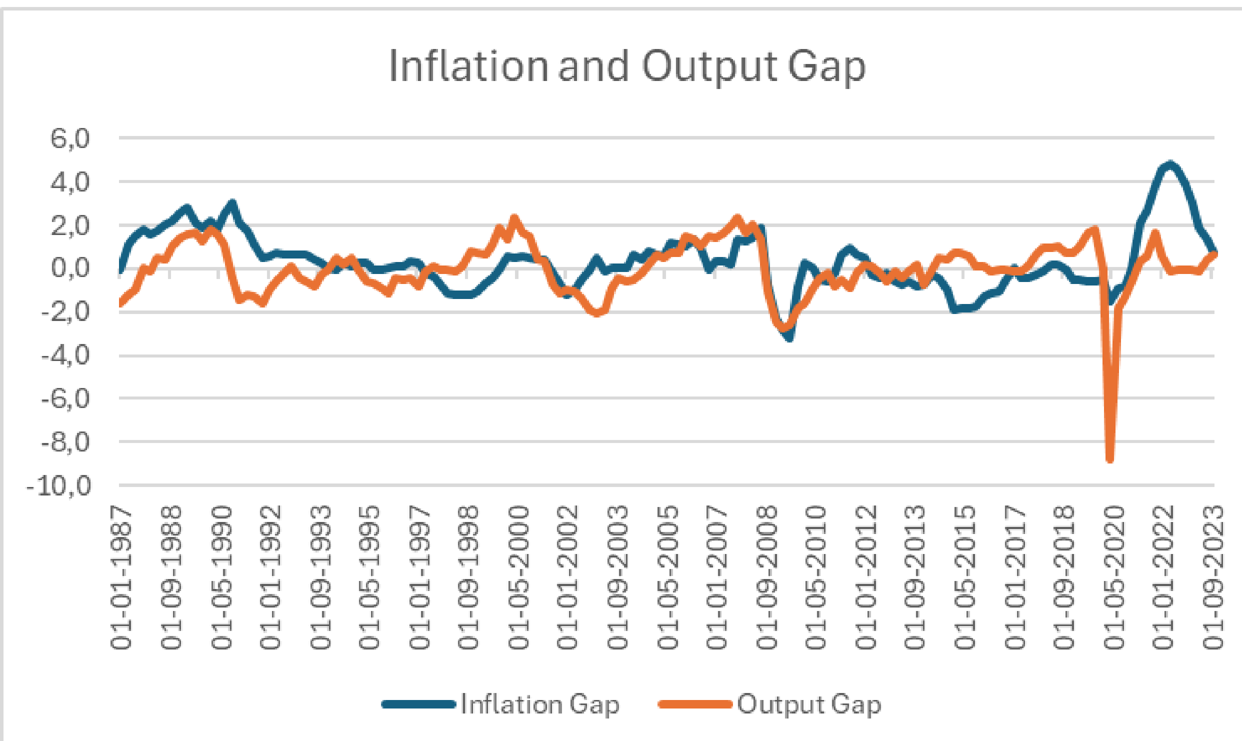
**Influences of $b$:**
**Negatively on $h$:** $h$ represents the central bank's responsiveness to inflation. A higher $h$ indicates a more vigorous reaction to inflation deviations. This stronger focus on controlling inflation means the central bank would prefer a smaller $b$ to avoid overreacting to output fluctuations, balancing between stabilizing inflation and output.
**Negatively on $\sigma_\epsilon^2$:** Higher measurement error variance ($\sigma_\epsilon^2$) suggests greater uncertainty

in the output gap measurements. With increased uncertainty, the central bank should react more cautiously to avoid overreacting to potentially noisy data. Therefore, the optimal $b$ decreases as $\sigma_\epsilon^2$ increases.
**Positively on $\sigma_\rho^2$:** $\sigma_\rho^2$ reflects the variance in nominal interest rate changes beyond the central bank's control. Greater variance implies larger uncontrollable changes in the interest rate, requiring the central bank to adjust more aggressively to stabilize the economy. Therefore, a higher $\sigma_\rho^2$ leads to a higher optimal $b$.
**Positively on $\sigma_v^2$:** The variance of shocks to consumer and business confidence ($\sigma_v^2$) affects the output gap directly. Larger variances in these shocks suggest more significant fluctuations in output, necessitating a stronger response from the central bank to maintain stability. And therefore $b$ increases with $\sigma_v^2$.

# 2 Problem 2

## 2.1

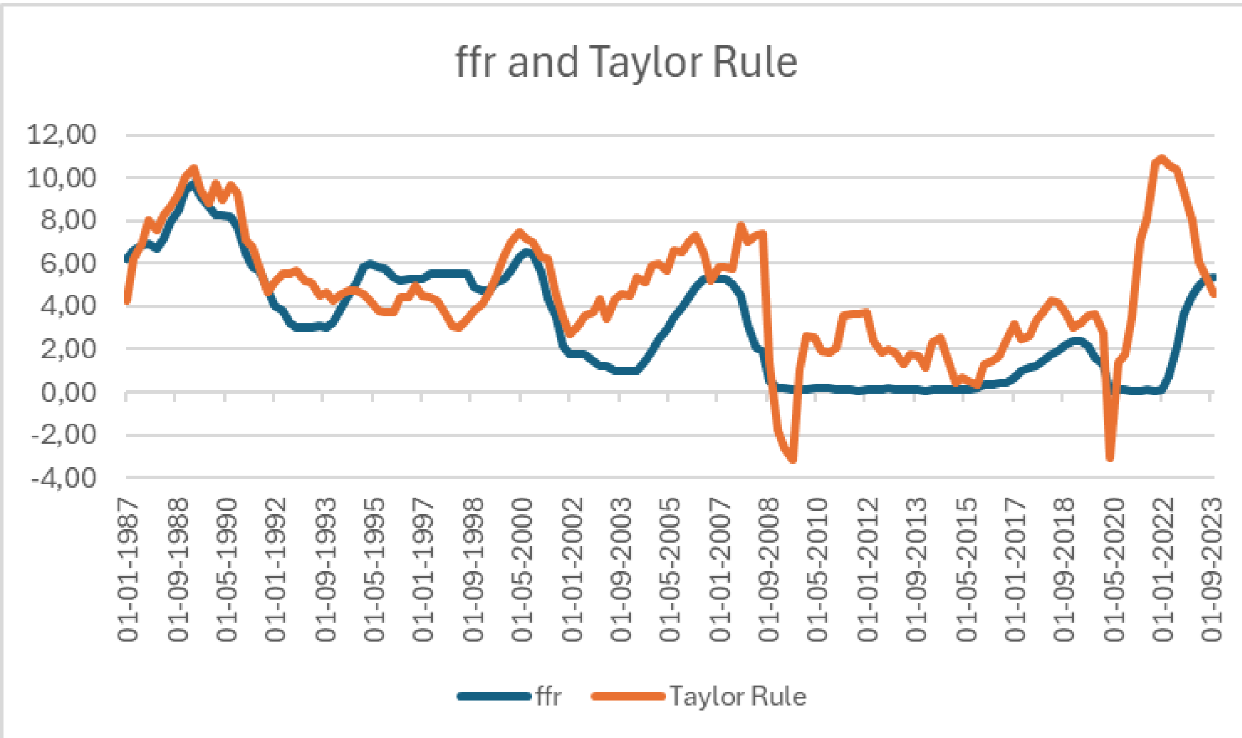
Calculating the inflation- and output gaps. For the Output gap the HP filter is used with a $\lambda = 1600$, since that, according to the book, is customary with quarterly data. The gaps are plotted below in figure 4.




Figur 4: Inflation and Output Gap

## 2.2

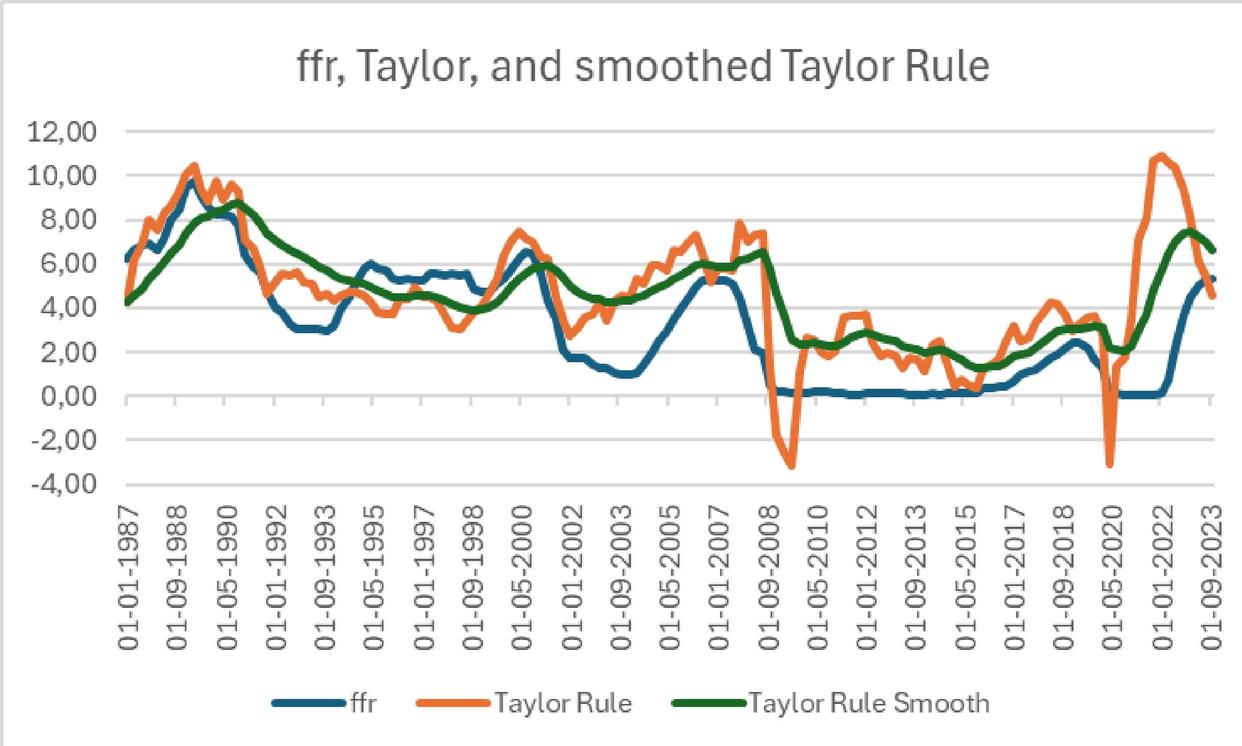
Calculating the Taylor rule for the economy, and plotting it below in figure 5. Here we see that the curve rarely follows it perfectly, but is quite well correlated with it. But in general, the ffr is smooth and does not make extreme jumps as much as the Taylor rule depicts.




Figur 5: Taylor rule based on the US economy, using $b = h = 0.5$

## 2.3

Calculating the Taylor rule with a smoothness factor using $\rho = 0.85$ is plotted below with the ffr and the normal Taylor rule in figure 6.




Figur 6: The ffr, Taylor rule, and the smoothed Taylor rule.

### 2.4

Looking at the graphs in questions 1, 2, and 3 we can see that the normal Taylor rule responds more swiftly to gaps in inflation or output, thereby more rapidly steering the economy back towards a potential equilibrium. This pattern is clear during the economic crisis of 2008 and the COVID-19 pandemic in 2020. However, this rapid response induces significant fluctuations in the nominal interest rate, which can be confusing and unsettling for consumers.

In contrast, the smoothed Taylor rule responds to longer-term gaps but does not react instantaneously or as severely to short-term or single-instance inflation/output gaps. This smoother adjustment curve is more advantageous for consumers who might find the big fluctuations unsettling. But it will then also come back slower to a potential equilibrium.

## 3 Problem 3

### 3.1

Equation 15, known as the Phillips curve, specifies that the inflation rate at time $t$ is determined by the expected inflation from the previous period plus $\gamma$ times the output gap.

This indicates that $\gamma$ represents the sensitivity of inflation to the output gap. Additionally, inflation in this model is influenced by potential supply shocks, denoted by $s_t$.

Equation 16, known as the IS curve, indicates that the output gap is positively affected by gaps in government spending, negatively affected by gaps in the nominal interest rate, and negatively affected by gaps in the tax rate. The parameters $\alpha_1$, $\alpha_2$, and $\alpha_3$ define their respective sensitivities. Additionally, the output gap is positively influenced by demand shocks $v_t$.

Equation 17, known as the Taylor rule, defines how the central bank sets its nominal interest rate to stabilize the economy. The baseline for this rule is the natural interest rate. However, when the economy is out of equilibrium, the nominal interest rate is influenced by the expected interest rate for the next period, possible output gaps, and deviations of inflation from its target. The nominal interest rate is also affected by $\hat{\rho}_t$, which represents a deviation from the Taylor rule or a potential shock. The parameters $b$ and $h$ are set by the central bank to determine how aggressively it reacts to output and inflation gaps.

Equation 18 defines the real interest rate as the nominal interest rate minus the expected inflation for the next period.

Equation 19 defines the expected inflation rate using an adaptive expectations model. In this model, the expected inflation is a weighted average, represented by $\phi$, between the expected inflation from the previous period and the observed inflation. The parameter $\phi$ determines the emphasis agents place on the expected versus observed inflation from the last period. If $\phi = 0$, agents rely solely on the observed inflation, $\pi$. If $\phi > 0$, agents also take into account their inflation expectations from the previous period.

## 3.2

For the AD curve, we know that $\hat{y}_t = y_t - \bar{y}$ and $\hat{\pi}_t = \pi_t - \bar{\pi}$, thus we can simply substitute that in:

$$y_t - \bar{y} = -a\left(\pi_t - \pi^*\right) + z_t \iff \hat{y}_t = -a\hat{\pi}_t + z_t$$

For the AS curve we first find $\pi_{t-1}^e$ using equation 15:

$$\pi_{t-1} = \pi_{t-1}^e + \gamma\left(y_{t-1} - \bar{y}\right) + s_{t-1} \iff \pi_{t-1}^e = \pi_{t-1} - \gamma\hat{y}_{t-1} + s_{t-1}$$

We then start by substituting equation 19 into 15, and then using the $\pi_{t-1}^e$ we just found:

$$\pi_t = \pi_t^e + \gamma\hat{y}_t + s_t \iff \pi_t = \phi\pi_{t-1}^e + (1-\phi)\pi_{t-1} + \gamma\hat{y}_t + s_t$$

$$\iff \pi_t = \phi(\pi_{t-1} - \gamma\hat{y}_{t-1} + s_{t-1}) + (1-\phi)\pi_{t-1} + \gamma\hat{y}_t + s_t$$

Now we just subtract $\pi^*$ from both sides and simplify:

$$\hat{\pi}_t = \phi(\pi_{t-1} - \gamma\hat{y}_{t-1} + s_{t-1}) + (1-\phi)\pi_{t-1} + \gamma\hat{y}_t + s_t - \pi^*$$

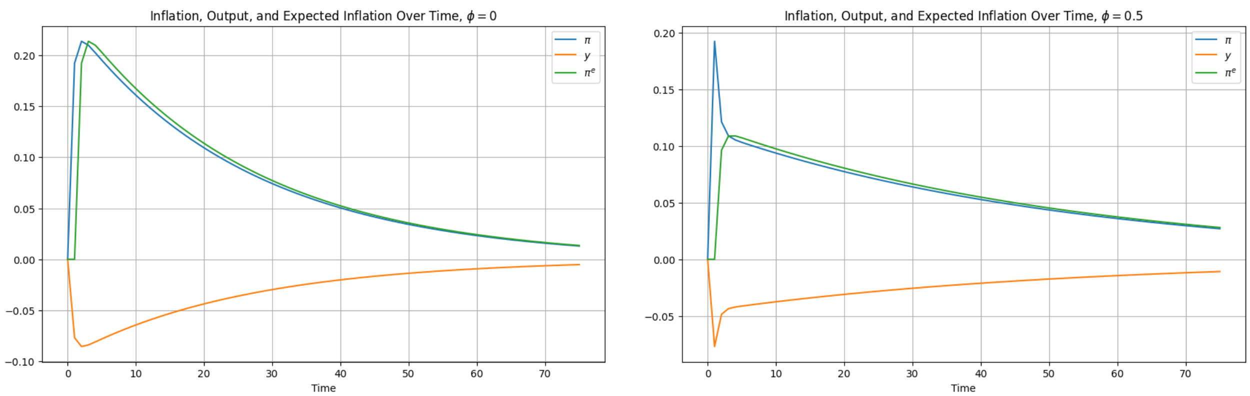
$$\iff \hat{\pi}_t = \hat{\pi}_{t-1} + \gamma\hat{y}_t - \phi\gamma\hat{y}_{t-1} + s_t - \phi s_{t-1}$$

## 3.3

Defining equations 22 and 23 in the notebook

## 3.4

Simulating a one-time negative supply shock, and plotting the impulse response functions below in figure 7.

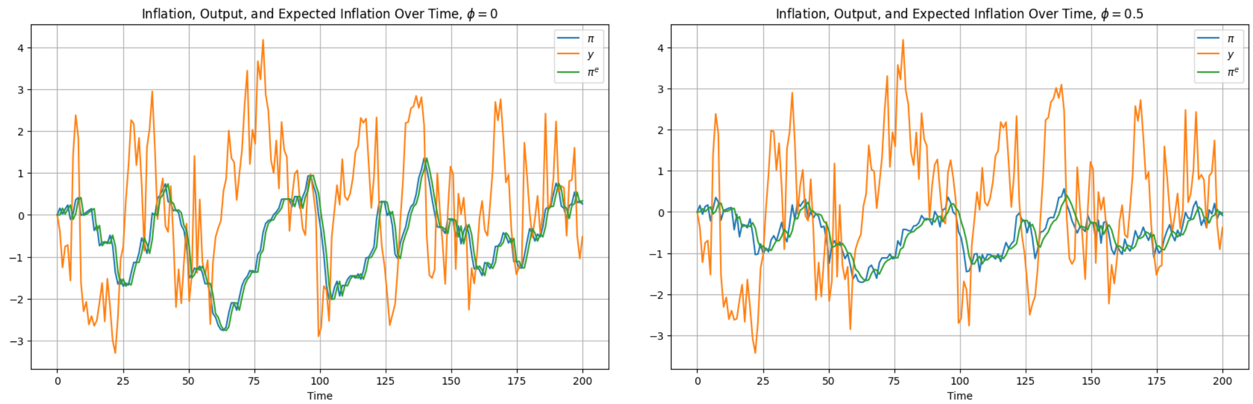



Figur 7: Inflation, output, and expected inflation simulated with a one-time negative supply shock over 75 periods.

When $\phi = 0.5$, the expectations place a greater emphasis on the expectations from the previous period, which makes the response to inflation shocks less pronounced. This is because the expected inflation incorporates past expectations, dampening the immediate impact of a one-time shock. When $\phi = 0$, all emphasis is placed on the observed inflation from the last period. As a result, expected inflation responds more strongly to a one-time shock to output, as it immediately adjusts to the last period's observed inflation. This leads to a larger increase in expected inflation and a longer period before the output gap closes.

## 3.5

Simulating the model for 200 periods with continuous random shocks as defined in the model using a random seed of "2023". The simulation is plotted below in figure 8. The statistics of the simulation can be shown below in table 1.




Figur 8: Inflation, output, and expected inflation simulated over 200 periods.

| **Statistic** | $\boldsymbol{\phi = 0}$ | $\boldsymbol{\phi = 0.5}$ |
|---|---|---|
| std($\hat{y}_t$) | 1.5828 | 1.5958 |
| std($\hat{\pi}_t$) | 0.8697 | 0.4987 |
| corr($\hat{y}_t, \hat{\pi}_t$) | 0.0604 | 0.1668 |
| corr($\hat{y}_t, \hat{y}_{t-1}$) | 0.7604 | 0.7685 |
| corr($\hat{\pi}_t, \hat{\pi}_{t-1}$) | 0.9565 | 0.8750 |

Tabel 1: Statistical measures for different values of $\phi$

## 3.6

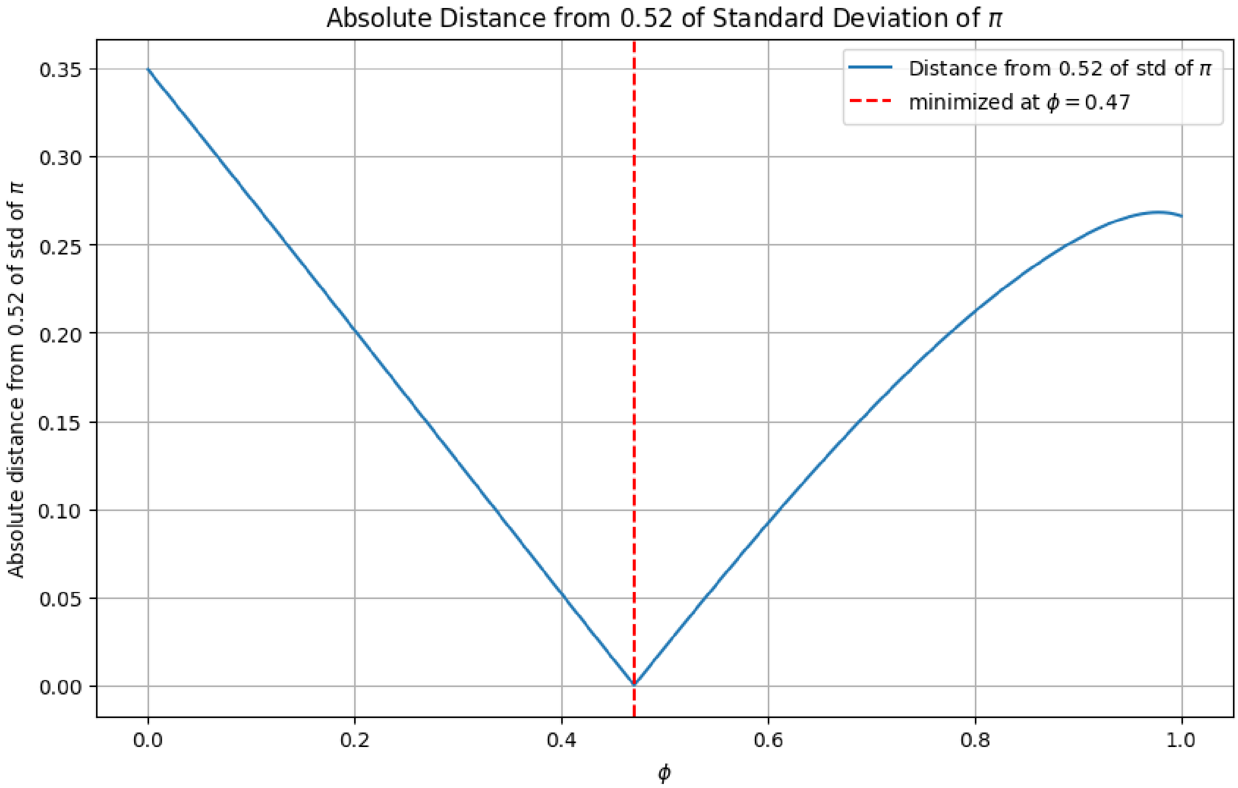
Computing the total social loss in the economy for when $\phi = 0$ and $\phi = 0.5$, and finding:

$$\mathbb{L}_{\phi=0} = 95.8671 \text{ and } \mathbb{L}_{\phi=0.5} = 86.4777$$

Thus we see a decrease in the total social loss when $\phi = 0.5$ compared to when $\phi = 0$.

## 3.7

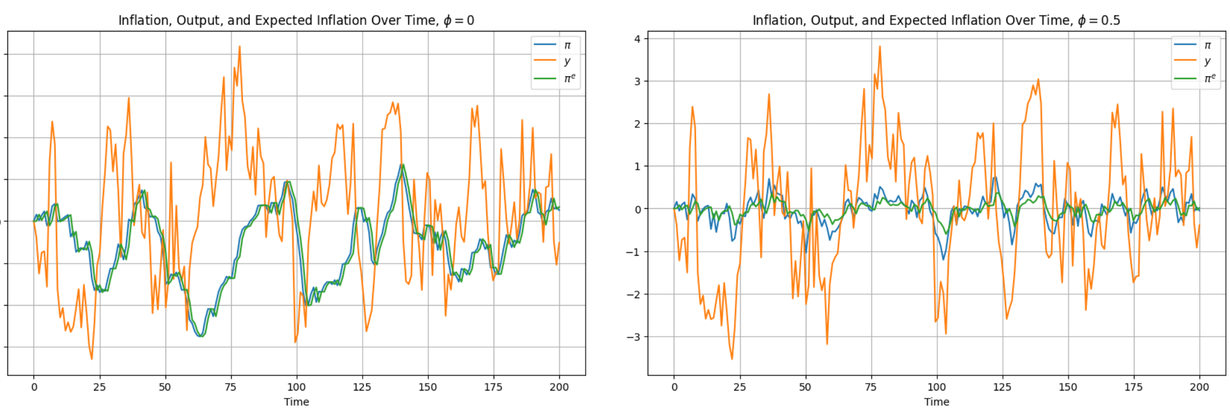
Calibrating the value of $\phi$, such that the simulated standard deviation of the inflation gap is 0.52. Thus we look for a $\phi$ that minimizes $|0.52 - \text{std}(\hat{\pi}_t)|$. This is found to be at $\phi = 0.4707$, and can be seen visually below in figure 9.




Figur 9: Finding the $\phi$ that minimizes the distance.

### 3.8

Performing the same simulation as in question 5, but using the newly defined model. The simulation can be seen below in 10. The computed statistics as well as the total social loss can be seen in table 2 together with the old model statistics for easier comparison.




Figur 10: Inflation, output, and expected inflation simulated over 200 periods with the new model.

| | Old Model | | New Model | |
|---|---|---|---|---|
| **Statistic** | $\phi = 0$ | $\phi = 0.5$ | $\phi = 0$ | $\phi = 0.5$ |
| std($\hat{y}_t$) | 1.5828 | 1.5958 | 1.5828 | 1.5443 |
| std($\hat{\pi}_t$) | 0.8697 | 0.4987 | 0.8697 | 0.3414 |
| corr($\hat{y}_t, \hat{\pi}_t$) | 0.0604 | 0.1668 | 0.0604 | 0.6856 |
| corr($\hat{y}_t, \hat{y}_{t-1}$) | 0.7604 | 0.7685 | 0.7604 | 0.7562 |
| corr($\hat{\pi}_t, \hat{\pi}_{t-1}$) | 0.9565 | 0.8750 | 0.9565 | 0.7067 |
| $\mathbb{L}$ | 95.8671 | 86.4777 | 95.8671 | 80.1704 |

Tabel 2: Statistical measures for different values of $\phi$ in the old and new models

Unsurprisingly we see that for $\phi = 0$ are they identical since they just collapse to the same expression. But for $\phi = 0.5$ we have a difference in the inflation, as we see to be much more volatile for the model using equations 22 and 23 has a much more volatile.

Equations 25, 26, and 27 incorporate a model where expected inflation is a weighted average of the central bank's inflation target and the previous period's inflation. This approach anchors expectations more firmly to the target, reducing the sensitivity of expected inflation to short-term deviations. As a result, the inflation rate exhibits less volatility because expectations are less responsive to shocks, leading to more stable inflation.

Thus, the key difference lies in how expectations are formed and incorporated into the models, with the first model (Equations 22/23) leading to more volatile inflation due to adaptive expectations, while the new model (Equations 25/26/27) results in more stable inflation due to the inclusion of the central bank's target in the expectations.

Furthermore, when comparing the social loss between the two models, we observe that the first model yields a social loss of 86.4777, whereas the new model results in a lower social loss of 80.1704. This outcome is intuitively reasonable, as the social loss function aims to minimize deviations from the output and inflation gaps. And as we see from the computed statistics, the new model shows lower standard deviations for both the output and inflation gap.

Thus seen from a perspective of minimizing the total social loss, the new model using equation 25/26/27 is more favorable.

## 3.9

The findings in this problem support Jerome Powell's statement. When a supply shock occurs, whether it is a one-time or a longer shock, the expectations in the first model show much higher volatility. As discussed in Question 8, this increased volatility results in a higher social loss. In contrast, the second model, which incorporates a way to align expectations with the central bank's target inflation, shows a more stable inflation and therefore a lower social loss. This stability shows the importance of anchoring inflation expectations to the central bank's target and thus helps avoid a de-anchoring of expectations.